Quantified possessives and direct compositionality

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1. Introduction

Direct compositionality is the hypothesis that semantic interpretation tightly follows syntactic combination. An important constraint placed by direct compositionality on the syntax-semantics interface is that every constituent generated by syntactic rules receives a model theoretic interpretation. Direct compositionality as a program and an empirical hypothesis has been a core topic in semantic theory for four decades now (Montague 1970, Jacobson 1999, 2000, Szabolcsi 2003, *inter alia*. A state of the art overview of direct compositionality as well as a collection of recent studies are found in Barker and Jacobson 2007). This study examined the issues raised for direct compositionality by the semantics of quantified possessive NPs like the one in (1), henceforth *quantified possessives*.

(1) **Every man's problems** were solved.

The various semantic and pragmatic problems raised by quantified possessives were first discussed and given an explicit analysis in Barker (1995). In that analysis, quantified possessives involve unselective quantification over cases. For example, (1) receives a logical form along the lines of (2), where R is the possessive relation.

(2) $every[man(x)\ \&\ problem(y)\ \&\ R(x,y)][solved(y)]$

This analysis is thus not directly compositional. For example, the syntactic constituent *every man* does not receive a semantic interpretation. More recently, Peters and Westerståhl (2006) (PW) have provided a thorough discussion of quantification in possessives. Their analysis diverges from Barker's in several significant ways. For example, it does not involve unselective binding, and it associates an interpretation with genitive *'s*. However, PW's analysis is also not directly compositional. Specifically, their analysis of quantified possessives with partitives such as (3) makes use of a non-directly compositional rule.

(3) Two of everyone's problems were solved.

The question therefore arises what a directly compositional analysis of quantified possessives might look like. The main aim of this study is to show that this is a non-trivial question. The only directly compositional analysis available in the literature, that of Barker (2005), is silent about partitives. It is also silent about a property

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of quantified possessives identified and discussed by PW, namely that they exhibit *variable quantificational force*. This property is discussed in detail in §2 below. After considering some possible extensions of Barker's (2005) and potential problems with them, I outline a directly compositional version of PW's analysis. This analysis turns out to require employment of the semantic operation of *pseudo-application*, invoked in Pratt and Francez (2001) for the analysis of quantified temporal modifiers. I discuss the possibility that the appearance of pseudo-application in these contexts is not a mere artifice of a convoluted formal analysis but reflects a basic property of contextual modification.

2. The meaning of quantified possessives

Consider the sentence in (1). Intuitively, this sentence asserts that everyone who has any problem is such that *all* of his or her problems were solved. The truth conditions of (1) can be captured by a representation like (4). The restriction to problem-owners is what Barker (1995) called *narrowing*. In the rest of this paper, I ignore narrowing.

(4) $\forall x(person(x) \;\&\; \exists y[problem(y) \;\&\; have(x,y)] \rightarrow$
$\qquad \forall y : have(x,y) \;\&\; problem(y) \rightarrow solved(y))$

More generally, abstracting away from certain details, PW show that the meaning of a quantified possessive can be represented as in (5), where Q_1 is a $\langle 1 \rangle$ quantifier (set of sets), Q_2 is a $\langle 1,1 \rangle$ quantifier (relation between sets), R is the possessive relation, taken to be an underspecified binary relation, P,Q are predicates, $dom_P(R)$ is the set of individuals who have an R-successor in (the extension of) P, and R_a is the set of a's R-successors.

(5) $Q_1(dom_P(R) \cap \{a : Q_2(P \cap R_a, Q)\})$

(6) exemplifies the representation of (1) above in these terms. In (6), the generalized quantifier *every man* is applied to the set of all individuals that have problems and whose every problem is solved. (Recall that narrowing is ignored here.)

(6) $every\_man(dom_{problem}(R) \cap \{a : every(problem \cap \{b : R(a,b)\}, solved)\})$

The observations made by PW which are the focus of this paper have to do with the interpretation of Q_2, the quantifier quantifying over the things possessed. Specifically, PW observe that Q_2 shows variable quantificational force, and that the number of Q_2s proliferates with iteration. These two observations are described below.

Variable quantificational force

First, when Q_2 is implicit, its force seems to be determined by context. Thus, (7a) is normally interpreted to assert that all of John's children are adopted, not that e.g. one out of his three children is. (7b) on the other hand can be used to say that several, perhaps many, of John's teeth are rotten, but not necessarily all of them. Similarly, (7c), from PW, can be used to claim that for most cars, at least one of their tires was slashed.

(7) a. John's children are adopted.

b. John's teeth are rotten.

c. Most cars' tires were slashed.

PW therefore make the value of Q_2 a contextual parameter.

The value of Q_2 is sometimes strongly influenced, if not determined, by the grammatical environment. For example, in the antecedent of a conditional (8)[1] and when the quantified possessor phrase is downward entailing (9), Q_2 is existential.

(8) If John's dogs escape he will be arrested. $(Q_2 = \exists)$

(9) Noone's dreams came true. $(Q_2 = \exists)$

It is worth pointing out that variable force in quantified possessives is strikingly similar to the well known problem of variable readings of donkey pronouns (Chierchia 1992, Kanazawa 1994, Lappin and Francez 1994, Yoon 1996, Merchant and Giannakidou 1998, Geurts 2002, Francez 2009, *inter alia*). The restriction to existential readings with downward monotone quantifiers in (9) is also shared by the two constructions. Exploring this affinity is beyond the scope of this paper. However, if the two cases of variable force are examples of a single phenomenon, then a unified analysis becomes a crucial desideratum, significantly constraining how both quantified possessives and donkey pronouns are analyzed.

Second, the value of Q_2 can be provided explicitly by a partitive phrase, as in (10). Here, the value of Q_2 is set to be that of the determiner preceding *of*.

(10) a. Two of everyone's dreams came true. (Q_2 = two)

b. Most of everyone's dreams came true. (Q_2 = most)

Iteration

It is well known that possessives can iterate, as in (11).

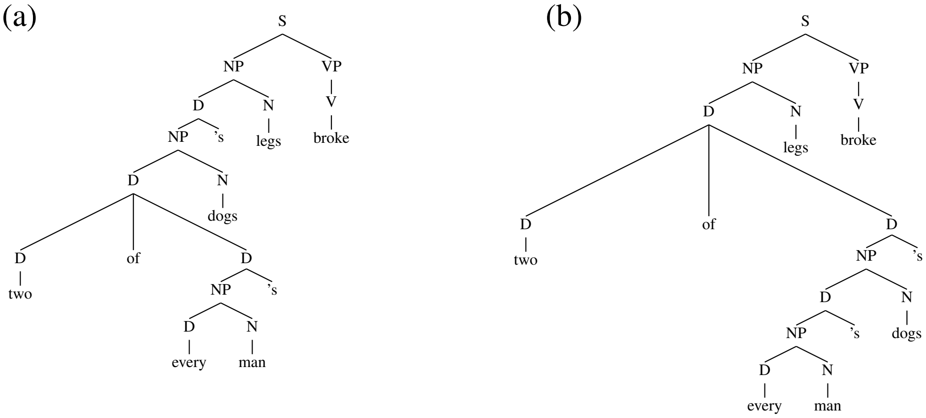
(11) Two of every man's dogs' legs broke.

(11) is syntactically ambiguous. The partitive *two of* can be read as modifying the possessive NP *every man's dogs' legs*, or it can be read as modifying the possessive NP *every man's dogs*. This syntactic ambiguity corresponds to an interpretational difference. In the first case, *two of* counts the number of legs, in the second the number of dogs. This is represented in (12), following PW's syntactic assumptions.[2]

[1] This kind of example was brought to my attention by Stanley Peters)

[2] One of these syntactic assumptions is that partitives form a constituent with a determiner rather than a full NP. This assumption is controversial, but seems to me to be semantically inconsequential.

(12) (a) (b)



The corresponding readings are represented in (13) and (14) respectively, with the contextually determined quantifier in both cases interpreted universally.

(13) $$every\_man(dom_{dog}(R) \cap \{a : \underline{\text{two}(dog} \cap R_a, \\ dom_{leg}(R) \cap \{c : \text{every}(leg \cap R_c, broke)\})\})$$

(14) $$every\_man(dom_{dog}(R) \cap \{a : \underline{every(dog} \cap R_a, \\ dom_{leg}(R) \cap \{c : two(leg \cap R_c, broke)\})\})$$

The important point here is that whenever a new possessed N is introduced, an implicit quantifier over the elements in the denotation of N possessed by the possessor is introduced with it. Let Q_{psd} stand for such quantifiers over possessed entities. A partitive can only contribute the value of that Q_{psd}associated with the N that is the immediate sister of the (complex) determiner of which the partitive is a constituent. Thus, *two of* must quantify over dogs in the structure in (12a), and over legs in the structure in (12b). Crucially, *two of* cannot quantify over dogs in (12b).

In summary, any analysis of quantificational possessives must capture the following descriptive generalizations:

- The meaning of quantified possessives involves quantification (Q_{psd}) over the possessed entities.
- The value of Q_{psd} can be determined by:
  - Context
  - Grammatical environment
  - Partitives
- A partitive inside a possessive determiner D can only contribute the value of the Q_{psd}restricted by the immediate N sister of D.

The next section describes a version of PW's analysis of quantified possessives, and points out where and how a directly compositional one must differ. Then the directly compositional analysis of Barker (2005) is discussed, and is argued not to capture the variability of quantificational force discussed above. An alternative proposal is outlined in §5, which makes use of an operation of *pseudo-application*. The nature of this operation and the light it potentially sheds on the grammar of modification are discussed in §5.1. §6 concludes with a general discussion and evaluation of the source and significance of the problems described.

3. A non-directly compositional analysis

Abstracting away from certain details, PW define an operation *poss* as in (15), which determines a determiner meaning, i.e. a $\langle 1,1\rangle$ quantifier. In (15), Q is a $\langle 1\rangle$ quantifier, Q_2 a $\langle 1,1\rangle$ quantifier, R the possessive relation, A,B any sets.

(15) $poss(Q,Q_2,R)(A,B) \leftrightarrow Q(dom_A(R) \cap \{a : Q_2(R_a \cap A,B)\}$

Following this definition, the denotation of possessive *'s* is given in (16). I use $\mathscr{Q}$ as a variable over $\langle 1\rangle$ quantifiers, and D as a variable over $\langle 1,1\rangle$ quantifiers (i.e. as a variable over Q_{psd}'s)

(16) $[\![\text{'s}\,]\!] = \lambda \mathscr{Q}_{(et,t)} \lambda P_{(et)} \lambda Q_{(et)}.\mathscr{Q}(dom_P(R) \cap \{a : D(R_a \cap P,Q)\})$

The meaning of a quantified possessive NP such as *every man's dog* is then composed as in (17).

(17) a. $[\![\text{every man's}]\!] = [\![\text{'s}]\!]([\![\text{every man}]\!]) =$
$\lambda \mathscr{Q}_{(et,t)} \lambda P_{(et)} \lambda Q_{(et)}.\mathscr{Q}(dom_P(R) \cap \{a : D(R_a \cap P,Q)\})(\lambda P.man \subseteq P) =$
$\lambda P_{(et)} \lambda Q_{(et)}.man \subseteq (dom_P(R) \cap \{a : D(R_a \cap P,Q)\})$
b. $[\![\text{every man's dog}]\!] = [\![\text{every man's}]\!]([\![dog]\!]) =$
$\lambda Q_{(et)}.man \subseteq (dom_{dog}(R) \cap \{a : D(R_a \cap dog,Q)\})$

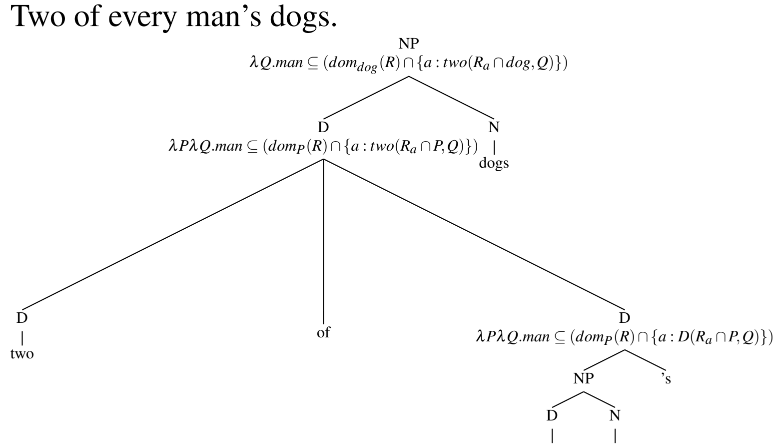
The meaning of the NP would be a generalized quantifier, but for the free variable D. It is thus a function from sets to open sentences, not to propositions, and so cannot be said to be the characteristic function of any set of sets. Rather, this meaning is a function from sets to a function from assignments of values to free variables to truth values. In PW's analysis, the value of D is a contextual parameter.

Partitives can be derived by the rule in (33), similar to PW's *P-rule* (p. 272)

(18) P-rule: An expression of the form [D1 of D2] where D2 is a possessive determiner of the form [NP 's] is interpreted as $\lambda D.[\![D2]\!]([\![D1]\!])$.

The derivation of the NP *two of every man's dogs* is shown in (19).

(19) Two of every man's dogs.



The interaction of iteration with partitives brings about a complication for compositionality not discussed by PW.[3] As mentioned, a partitive may only bind

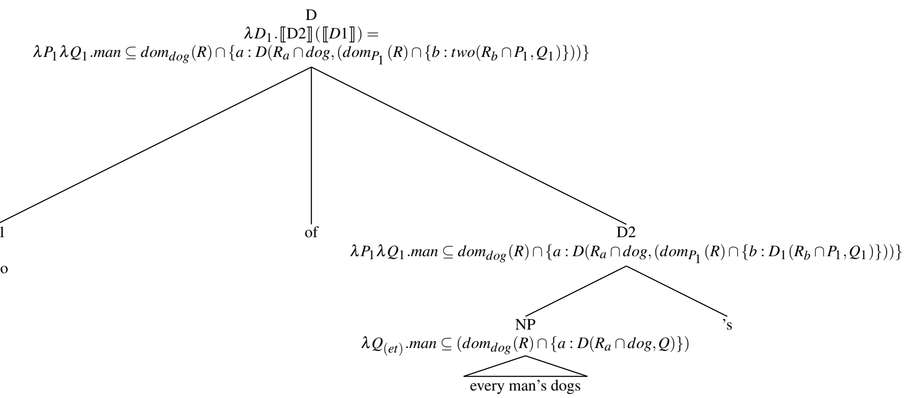
[3] This problem in fact does not arise for PW, because their rule for partitive formation can "see" into the derivation of the complement to *of*.

the variable for the quantifier quantifying over the elements in the extension of the N that is the sister of the complex determiner of which the partitive is part. At the point in the derivation at which the partitive composes, however, the meaning of the determiner in the complement of *of* might already contain any number of free variables over Q_{psd}'s that were introduced by preceding occurences of *'s*. An example of iteration in combination with a partitive is given in (20).

(20) Two of every man's dogs' legs broke.

(21) shows the derivation of the reading in which the partitive combines with the determiner *every man's dog's*.

(21)



In (21), *two* provides the value of the variable D_1, and not D, because it is D_1 that is abstracted over. However, nothing in the way the P-rule in (18) is formulated ensures this. Modifying the P-rule so as to ensure that the right variable is abstracted over requires some mechanism of allowing this rule access to the derivational history of the determiner complement of *of*.[4] Presumably, such a mechanism can be defined (though this does not seem trivial), but exploring such a mechanism is beyond the scope of this paper. The important point here is that, even given such a mechanism, the analysis just sketched is not directly compositional. First, because *of* does not receive any model theoretic interpretation. Second, because the operation of abstraction over a free variable does not correspond to any model theoretic operation.

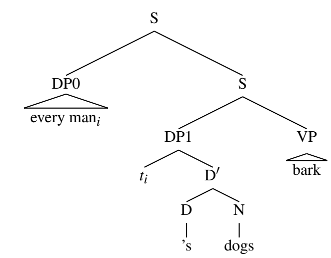
Before moving on to discuss the directly compositional analysis in Barker (2005), I briefly point out that capturing the interaction of partitives, iteration and variable quantificational force is a challenge also for an analysis assuming a syntactic level of LF. For example, a likely LF representation for a simple sentence like (22) is as in (23).[5]

(22) Every man's dogs bark.

[4] Notice that analyzing *two of* as taking an NP complement rather than a determiner complement does not affect this point.

[5] This is not the representation in Barker (1995). I preclude discussion of that analysis here for reasons of space, and since it is not clear to me how that analysis should be extended to deal with partitives.

(23)

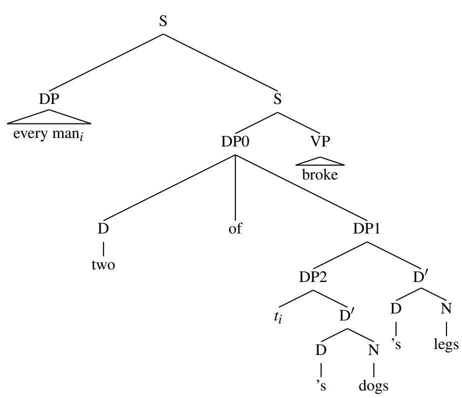


This LF can be translated informally as in (24):

(24) $every\_man(\lambda x.x\text{'s } dogs\ bark)$

It is natural to assume that the expression *x's dogs* denotes a plural individual, say the sum of all of x's dogs. If this is the case, then a possessive DP like DP1 in (23) denotes an individual. Already a question arises as to how variable quantificational force is to be captured in such an analysis. This question is discussed in the next section, but suppose for the moment it can be ignored. Given the assumptions so far, a natural LF for the more complex (20) is (25)

(25)



If possessive DPs denote plural individuals, then so does DP1 in (25). This seems an attractive assumption to make since it allows a simple analysis of partitives along the lines of Ladusaw (1982). Thus, *of* could be assigned a denotation mapping a plural individual into the set of its atomic elements, and this set could in turn function as the restrictor for the determiner *two*. However, DP2 on this analysis must also be assumed to denote a plural individual, and this assumption leads to rather absurd truth conditions. If DP2 denotes the individual consisting of all of x_i's dogs, then DP2 denotes the individual consisting of all the legs belonging to the sum of all of x_i's dogs. Of course, under natural assumptions, there is no leg that belongs to this sum individual, and even if there were, the sentence in (20) does not require for truth that anyone have dogs that share any legs. The upshot of this discussion is that the problems raised by the data discussed so far are not obviated by an LF-based analysis, which would have to introduce structural complications somewhere to accommodate them.

4. Barker 2005: Towards direct compositionality.

Barker (2005) outlines in brief a directly compositional analysis of quantified possessives. The analysis diverges in several respects from that of PW's. First, genitive *'s* is not taken to contribute any meaning. Instead, the possessive relation comes from the interpretation of the possessed noun. This is made possible by essentially treating all nouns as relational. A simple quantified possessive like (26) are interpreted as in (27), with the possessed noun treated as denoting an (e,e) function from individuals to their mothers.

(26) Everyone's mother

(27) a. $[\![\text{mother}]\!] = \lambda x.f_{mother}(x)$, f_{mother} is a function of type (e,e)

b. After **value raising** and **argument raising**[6]:
$\lambda \mathscr{P} \lambda P.\mathscr{P}(\lambda z.P(f_{mother}(z)))$

c. $[\![\text{everyone's mother}]\!] = \lambda P.everyone(\lambda x.P(f_{mother}(z)))$

Iteration is not discussed explicitly in Barker (2005). However, the logical form given to a sentence like (28) is give as the last line in (29). Presumably, since all nouns are relational, the meaning of *dog* is also taken to be an (e,e) function, mapping any individual to their dog, or, more generally, to the sum of all their dogs.

(28) Everyone's mother's dog left.

(29) a. $[\![\text{everyone's mother}]\!] = \lambda P.everyone(\lambda x.P(f_{mother}(x)))$

b. $[\![\text{dog}]\!] = \lambda \mathscr{P} \lambda Q.\mathscr{P}(\lambda x.Q(f_{dog}(x)))$, After *argument raising* and *value raising*.

c. $[\![\text{everyone's mother's dog}]\!] = [\![\text{dog}]\!]([\![\text{everyone's mother's}]\!]) =$
$\lambda Q.everyone(\lambda x.Q(f_{dog}(f_{mother}(x))))$,
i.e. map *dog* to an (e,e) function, mapping every individual to the sum of their dogs.

Barker (2005) is mostly concerned with the problems posed by quantified possessives for a variable free theory of binding. Therefore, he discusses neither variable quantificational force nor partitives. Here I discuss some possible ways to extend his analysis to cover these facts.

While variable force is not an issue with relational nouns like *mother*, of which there is only one for each individual (as long as *mother* is taken to mean "biological mother"), it does arise in the general case. Accommodating variable force raises the same problem for Barker's analysis as was mentioned earlier in the discussion of an LF-based analysis. The problem is that possessed nouns are ultimately non-quantificational but rather denote individuals.

One way of maintaining the sum-individual analysis and at the same time capturing variable force is to allow functions such as f_{dog} in (29) to choose sums

[6]Value raising and argument raising are two type shifting operations the details of which are not important here

other than the maximal sum of possessed entities. For example, for any individual a, it is possible to let the value of $f_{dog}(a)$ be a member in the set of sums of dogs belonging to a, rather than the maximal sum. Context can then determine which sum is selected. The universal reading is generated when the maximal sum is selected. Otherwise, the function f_{dog} selects some sum of dogs belonging to a, which gives rise to a reading approximating the existential reading.

Several objections may be raised against this line of analysis. One is that it models an utterance with a non-universally interpreted quantified possessive as asserting something about particular entities, namely the elements of the selected sum. Intuitively, this seems false. For example, (30) does not seem to make a claim about any particular dogs of John's.

(30) If John's dogs escape he will be sad.

Another is that when the possessor is a downward entailing quantifier, this analysis seems to generate the wrong truth conditions regardless of whether the sum-individual selected is maximal or not. Consider (31).

(31) No man's dogs were poisoned.

This sentence is true iff no man is such that any of his dogs were poisoned. However, on a sum-individual analysis, the representation of this sentence is as in (32).

(32) $no\_man(\lambda x.poisoned(f_{dogs}(x))$

If $f_{dogs}(x)$ denotes the maximal sum of x's dogs, then the sentence is wrongly predicted to assert that no man is such that all their dogs were poisoned. If $f_{dogs}(x)$ denotes, for any x, some non-maximal sum of x's dogs, then the sentence is wrongly predicted to assert that no man is such that some of his dogs were poisoned.

Now consider what an extension of this analysis to partitives might look like. On the assumption that possessed nouns are not quantificational but contribute sum-individuals, an extension to partitives might model them as contributing quantification over the atoms making up the sum individual. For example, a sentence like (33) could be given a logical form like (34), where $\leq$ is the part-whole relation.

(33) Two of every man's dogs bark.

(34) $every\_man(\lambda x.two(\{z : z \leq f_{dog}(x)\}, \{y : bark(y)\}))$

However, on these assumptions, it seems natural to assign (35), on one of its readings, the logical form in (36).

(35) Two of most women$_i$'s children's children knew them$_i$.

(36) $most\_women(\lambda x.two(\{z : z \leq f_{child}(f_{child}(x))\}, \{y : knew(y,x)\}))$

But this logical form runs into the same problem discussed earlier for an LF-based analysis with individual-sums. (36) requires for truth that the children of at least some women have a child together. The sentence in (35) does not have these incestual requirements for truth. It merely requires that most women are such that all of their children have at least two children who knew their grandmother.

Furthermore, assuming that the interpretation of possessed nouns varies between maximal and non-maximal sums, some method is required to control this variation in the presence of a partitive. Specifically, when a partitive is present, the selected sum must be constrained to be maximal. Otherwise, (33) is wrongly predicted to be true when John has twenty dogs only four of which bark, since in that case there is nevertheless a (non-maximal) sum of dogs belonging to John such that most of the atoms in that sum are dogs that bark.[7]

In summary, the analysis in Barker (2005) as is is silent about variable quantificational force and partitives. At least the extensions of this analysis outlined above (which are not by any means complete or exhaustive of the possibilities), still leave much of the picture unclear. In the following section, I suggest another alternative, which is essentially a slight modification of the analysis proposed by PW.

5. A directly compositional analysis with pseudo-application

The denotation for *'s* remains exactly the same as in (16) above, except that the free variable over Q_{psd}'s is bound rather than free. The types of variables are henceforth omitted.

(37) $[\![\text{'s}]\!] = \lambda \mathscr{Q} \lambda P \lambda Q \lambda D. \mathscr{Q}(\lambda x. dom_P(R)(x) \ \& \ D(R_x \cap P, Q)\})$

The meaning of a possessive determiner is thus as in (38). Note that the type of possessive determiners is

(38) $[\![\text{Everyone's}]\!] = \lambda P \lambda Q \lambda D. everyone(\lambda x. dom_P(R)(x) \ \& \ D(R_x \cap P, Q)\})$

Possessive determiners are functions from pairs of sets to functions from determiner meanings , Q_{psd}'s, to truth values (type $(et,(et,dt))$, where d is short for $(et,(et,t))$. A possessive NP on this approach denotes a function from sets to functions from Q_{psd}'s to truth values. Possessive sentences denote a function from Q_{psd}'s to propositions.

At this point a problem arises with iteration: *'s* now cannot combine directly with a possessive NP. The first argument of the function denoted by *'s* is a $\langle 1 \rangle$ quantifier, but the denotation of a possessive NP is not quite a generalized quantifier, but rather a function from sets to functions from Q_{psd}'s to truth values. In other words, λD in (39) is in the way.

(39) $[\![\text{Everyone's dogs}]\!] = \lambda Q \lambda D. everyone(\lambda x. dom_{dog}(R)(x) \ \& \ D(R_x \cap dog, Q))$

This is an instance of a more general siutation, discussed by Pratt and Francez (2001) in a different context. Pratt and Francez (henceforth PF) characterize the general case as follows. Given a variable x of some type τ, a variable Q of type (τ,t), and variables u,v of any type, the function in (40a) cannot apply to the argument in (40b) because of the intervening variable v

[7]I owe this observation to Stanley Peters.

(40) a. $\lambda Q_{\tau,t}\lambda u.\psi(Q,u)$

b. $\lambda x_{\tau}\lambda v.\phi(x,v)$

This is precisely the situation in (39). PF then define an operation, which they term *pseudo-application*, by which to combine the formula in (40a) with that in (40b). The operation is defined in (5.1). I use the notation $A\wr B\int$ to write that A is pseudo-applied to B.

(41) $\lambda Q\lambda u.\psi(Q,u)\wr\lambda x_{\tau}\lambda v.\phi(x,v)\int =_{def}\lambda u\lambda v.(\lambda Q.\psi(Q,u)(\lambda x.\phi(x,v))) = \lambda u\lambda v.\psi(\lambda x.\phi(x,v),u)$

The workings of this operation are easy to grasp by considering a sequence of operations familiar from the way quantifying-in is done in Montague Grammar, as outlined in (42).

(42) a. Apply the "offensive" formula in (39b) to two dummy arguments.
$\lambda x\lambda v.\phi(x,v)(x_1)(v_1) = \phi(x_1,v_1)$

b. Abstract over x_1, yielding $\lambda x_1.\phi(x_1,v_1)$

c. . Feed the result to the formula in (39b):
$\lambda Q\lambda u.\psi(Q,u)(\lambda x_1.\phi(x_1,v_1)) = \lambda u.\psi(\lambda x_1.\phi(x_1,v_1),u)$

d. Apply the result to u_1, yielding:
$\psi(\lambda x_1.\phi(x_1,v_1),u_1)$

e. Abstract over v_1:
$\lambda v_1.\psi(\lambda x_1.\phi(x_1,v_1),u_1)$

f. Abstract over u_1:
$\lambda u_1\lambda v_1.\psi(\lambda x_1.\phi(x_1,v_1),u_1)$

The final formula in (42) is equivalent to the result of pseudo-application in (5.1) above.

Quantified possessives can combine with *'s* by pseudo-application. The full derivation is shown in (43) for *every man's dogs*.

(43) a. $[\![\text{'s}]\!] = \lambda \mathscr{Q} \lambda P \lambda Q \lambda D.\mathscr{Q}(\lambda x.dom_P(R)(x) \;\&\; D(R_x \cap P, Q)\})$

b. $[\![\text{Every man's dogs}]\!] = \lambda Q_1 \lambda D_1.every\_man(\lambda z.dom_{dog}(R)(z) \;\&$
$D_1(R_z \cap dog, Q_1))$

c. $[\![\text{Every man's dogs'}]\!] = [\![\text{'s}]\!] \wr [\![\text{Every man's dogs}]\!] \int =$
$\lambda P \lambda Q \lambda D \lambda D_1.[\mathscr{Q}(\lambda x.dom_P(R)(x) \;\&\; D(R_x \cap P, Q))]$
$(\lambda Q_1.every\_man(\lambda z.dom_{dog}(R)(z) \;\&\; D_1(R_z \cap dog, Q_1))) =$
$\lambda P \lambda Q \lambda D \lambda D_1.[\lambda Q_1.every\_man(\lambda z.dom_{dog}(R)(z) \;\&\; D_1(R_z \cap dog, Q_1))]$
$(\lambda x.dom_P(R)(x) \;\&\; D(R_x \cap P, Q)) =$
$\lambda P \lambda Q \lambda D \lambda D_1.every\_man(\lambda z.dom_{dog}(R)(z) \;\&$
$D_1(R_z \cap dog, \lambda x.dom_P(R)(x) \;\&\; D(R_x \cap P, Q)))$

The meaning in (43) can be combined with a possessed noun to give the correct meaning for an iterated possessive NP such as *everyone's dogs' legs*. The result can than combine with a VP such as *barks* to form a possessive sentence, which is a function from Q_{psd}'s to propositions. The deeper the embedding, the more Q_{psd}'s are required to form a proposition.

The derivation of a partitive determiner is shown in (44), assuming the meaning for *of* in (44b), where q is a variable over possessive NPs. Note that possessive NPs do not have a unique type - their type depends on the depth of embedding involved. However, abbreviating the type of $\langle 1,1 \rangle$ quantifiers as d, their type will always be $(et,(et,(d,\ldots(dt))))$. This means that *of* must be polymorphic.

(44) a. Two of every man's

b. $[\![\text{of}]\!] = \lambda q \lambda D \lambda P \lambda Q.q(P)(Q)(D)$

c. $[\![\text{every man's}]\!] = \lambda P_1 \lambda Q_1 \lambda D_1.every\_man(\lambda x.dom_{P_1}(R)(x) \;\&$
$D(R_x \cap P_1, Q_1)\})$

d. $[\![\text{of every man's}]\!] = [\![\text{of}]\!]([\![\text{every man's dogs}]\!]) =$
$\lambda q \lambda D \lambda P \lambda Q.q(P)(Q)(D)$
$(\lambda P_1 \lambda Q_1 \lambda D_1.every\_man(\lambda x.dom_{dog}(R)(x) \;\&\; D_1(R_x \cap dog, Q)) =$
$\lambda D \lambda P \lambda Q.every\_man(\lambda x.dom_P(R)(x) \;\&\; D(R_x \cap P, Q))$

e. $[\![\text{two of every man's}]\!] = [\![\text{of every man's}]\!]([\![\text{two}]\!]) =$
$\lambda P \lambda Q.every\_man(\lambda x.dom_P(R)(x) \;\&\; two(R_x \cap P, Q))$

Accounting for the interaction of partitives with iteration is now straightforward. The denotation of *of* ensures that it is always the outmost variable over Q_{psd}'s in the denotation of a possessive determiner that is something.

(45) a. Two of every man's dogs'

b. $[\![\text{every man's dogs'}]\!] = [\![{}'s]\!] \wr [\![\text{every man's dogs}]\!] \int =$
$\lambda P \lambda Q \lambda D \lambda D_1.every\_man(\lambda z.dom_{dog}(R)(z) \;\&$
$D_1(R_z \cap dog, \lambda x.dom_P(R)(x) \;\&\; D(R_x \cap P, Q)))$

c. $[\![\text{of every man's dogs'}]\!] = [\![of]\!]([\![\text{every man's dogs'}]\!]) =$
$\lambda q \lambda D_2 \lambda P_1 \lambda Q_1.q(P_1)(Q_1)(D_2)$
$(\lambda P \lambda Q \lambda D \lambda D_1.every\_man(\lambda z.dom_{dog}(R)(z) \;\&$
$D_1(R_z \cap dog, \lambda x.dom_P(R)(x) \;\&\; D(R_x \cap P, Q))) =$

$\lambda D_2 \lambda P_1 \lambda Q_1 \lambda D_1.every\_man(\lambda z.dom_{dog}(R)(z)$ &
$D_1(R_z \cap dog, \lambda x.dom_{P_1}(R)(x)$ & $D_2(R_x \cap P_1, Q_1)))$

d. ⟦two of every man's dogs' =
$\lambda P_1 \lambda Q_1 \lambda D_1.every\_man(\lambda z.dom_{dog}(R)(z)$ &
$D_1(R_z \cap dog, \lambda x.dom_{P_1}(R)(x)$ & $two(R_x \cap P_1, Q_1)))$

This analysis thus captures the two main descriptive properties of quantified possessives brought to light by PW's analysis, namely variable quantificational force, the contribution of partitives and their interaction with iteration. The analysis is also directly compositional – all syntactic constituents are semantic constituents with a determiner model theoretic meaning.

5.1. Pseudo-application and the grammar of modification

The use of pseudo-application raises some questions. On the one hand, it might seem like an artificial feature of the analysis. On the other, the fact that this operation is useful in two *prima facie* unrelated context (the analysis of genitive *'s* and the analysis of temporal modifiers) might indicate that it plays some real role in grammar (at least in a directly compositional grammar) which should be elucidated. Here I attempt a preliminary characterization of what this role might be.

Pratt and Francez invoke pseudo-application in order to account for a property of temporal modifiers, they call *cascading*, and which is closely linked to the kind of iteration discussed here for possessives and its concomitant proliferation of Q_{psd}'s. Temporal modifiers can be "stacked" upon one another, as in (46).

(46) Brutus stabbed Caesar after every Senate meeting during the summer.

The full details of PF's analysis cannot be repeated here, but the important intuition is that the temporal modifier *during the summer* describes the temporal context within which the meetings quantified over in the first modifier take place. In order to capture this, PF assign to temporal nouns like *meeting* a relational meaning as in (47), where the function *time* maps any event to its running time. This denotation includes a variable *I* over intervals, which corresponds to the interval *within which* the event described by the common noun occurs.

(47) ⟦meeting⟧ $= \lambda x \lambda I.meeting(x)$ & $time(x) \subseteq I$

Pseudo-application is called for in order to combine determiners with such temporal nouns. In particular, it is required that the contextual interval variable *I* be accessible at the point in the derivation at which the second modifier, *during the summer*, is introduced, since the value of this variable is restricted by that modifier. In fact, the role of pseudo-application in PF's account is strikingly similar to its role in the account of quantified possessives described above. Pseudo-application is needed to apply a function to an argument that contains a contextual variable, the value of which can be contributed or restricted compositionally by explicit material introduced later on in the derivation. In the case of temporal modifiers, this is the contextual interval within which an event is situated. In the case of possessives, it is the contextual quantifier over the things possessed that is present in any quantificational NP (Q_{psd}). Since the material that explicitly contributes the value of this quantifier is introduced later in the derivation, when a partitive is introduced which modifies a determiner containing the relevant NP, access to this contextual variable must be available at the determiner level. Of course, it is possible that either PF's analysis of temporal modifiers, or the version of PW's analysis I propose here for quantified possessives, or both, are incorrect, and that pseudo-application is ultimately a side effect of misanalysis. Further research is required in order to determine this, but the very question is intriguing.

6. Summary and conclusions

This paper examined the question of what a directly compositional analysis of quantification in possessives might look like which captures some of the key observations in Peters and Westerståhl (2006). I argued that accounting for the full range of data – variable force, iteration and partitives – poses a challenge for both directly compositional and non-directly compositional analyses. A possible extension of the analysis outlined in Barker (2005) using (*ee*)-functions was considered, and was argued to be problematic. A directly compositional implementation of Peters and Westerståhl's analysis, recognizing the presence of an implicit quantifier over possessed entities, was proposed. On this analysis, the interaction of partitive *of*-phrases with iteration of possessives turned out to require invocation of pseudo-application, a mode of combination beyond application and composition. Interestingly, pseudo-application has been proposed in the literature (Pratt and Francez 2001) for the analysis of temporal modifiers. The relation between temporal modification and partitive-modification was briefly discussed. I suggested that the requirement for pseudo-application in both contexts might be non-accidental, reflecting an essential aspect of the semantics of modification, namely the fact that contextually determined information can, in some cases, be supplied compositionally at a non-local point in a derivation.

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