International Encyclopedia of Unified Science

Foundations
of the Unity of Science
(Volumes I-II of the Encyclopedia)

The University of Chicago Press. Chicago, Illinois

International Encyclopedia of Unified Science

Volume I · Number 3

Foundations of Logic and Mathematics

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C and thereby to change them from formulas to propositions. We proceed in the following way. It is already determined by the rules F which expressions are formulas in C. Now we have to stipulate that each of them is also a proposition in S. By the syntactical classification of the signs it is not yet completely settled which signs are logical and which descriptive. In many cases there is still a considerable amount of freedom of choice in this respect, as we shall see later in some examples. After having stated which signs are to be logical and which descriptive, we construct the rules SL for the logical signs. Here our choice is restricted to some extent by the requirement that the interpretation must be true.

Finally we establish the rules SD for the descriptive signs. Here we have to take into account the classification of signs. We choose the designata for each kind of signs and then for each sign of that kind. We may begin with individual names. First we choose a field of objects with which we wish to deal in the language to be constructed, e.g., the persons of a certain group, the towns of a certain country, the colors, geometrical structures, or whatever else. Then we determine for each individual name, as its designatum, one object of the class chosen. Then, for each predicate, we choose a possible property of those objects, etc. In this way, a designatum for every descriptive sign is chosen. If we decide to make S an L-true interpretation of C, we have a great amount of freedom for the choice of the rules SD. Otherwise, we find some essential restrictions. If some of the C-true formulas are to become factual propositions, they must be factually true. Therefore, in this case, on the basis of our factual knowledge about the objects which we have chosen as subject matter of Z, we have to take care that the interpretations for the descriptive names, predicates, etc., i.e., their designata, are chosen in such a way that those factual C-true sentences are actually true.

12. Is Logic a Matter of Convention?

There has been much controversial discussion recently on the question whether or not logic is conventional. Are the rules on

which logical deduction is based to be chosen at will and, hence, to be judged only with respect to convenience but not to correctness? Or is there a distinction between objectively right and objectively wrong systems so that in constructing a system of rules we are free only in relatively minor respects (as, e.g., the way of formulation) but bound in all essential respects? Obviously, the question discussed refers to the rules of an interpreted language, applicable for purposes of communication; nobody doubts that the rules of a pure calculus, without regard to any interpretation, can be chosen arbitrarily. On the basis of our former discussions we are in a position to answer the question. We found the possibility—which we called the second method—of constructing a language system in such a way that first a calculus C is established and then an interpretation is given by adding a semantical system S. Here we are free in choosing the rules of C. To be sure, the choice is not irrelevant; it depends upon C whether the interpretation can yield a rich language or only a poor one.

We may find that a calculus we have chosen yields a language which is too poor or which in some other respect seems unsuitable for the purpose we have in mind. But there is no question of a calculus being right or wrong, true or false. A true interpretation is possible for any given consistent calculus (and hence for any calculus of the usual kind, not containing rules for 'C-false'), however the rules may be chosen.

On the other hand, those who deny the conventional character of logic, i.e., the possibility of a free choice of the logical rules of deduction, are equally right in what they mean if not in what they say. They are right under a certain condition, which presumably is tacitly assumed. The condition is that the "meanings" of the logical signs are given before the rules of deduction are formulated. They would, for instance, insist that the rule R 1 of B-C ('from 'wenn . . . , so - - -' and '. . .', '- - -' is directly derivable' [§ 9]) is necessary; that it would be wrong to change it arbitrarily, e.g., into R 1*: 'from 'wenn . . ., so - - -' and 'nicht . . .', '- - -' is directly derivable'. What they presumably mean is that the rule R 1* is incorrect on the basis of

the presupposed "meaning" of the signs 'wenn', 'so', and 'nicht'. Thus they have in mind the procedure which we called the first method (§ 11): we begin by establishing the semantical rules SL or assume them as given—obviously this is meant by saying that the "meaning" is given—and then we ask what rules of deduction, i.e., syntactical rules of transformation, would be in accordance with the presupposed semantical rules. In this order of procedure, we are, as we have seen, indeed bound in the choice of the rules in all essential respects. Thus we come to a reconciliation of the opposing views. And it seems to me that an agreement should easily be attainable in the other direction as well. The anti-conventionalists would certainly not deny that the rule R 1* can also be chosen and can lead to correct results, provided we interpret the logical signs in a different way (in the example given, we could interpret 'wenn . . . , so - - -', e.g., as '. . . or - - -').

The result of our discussion is the following: logic or the rules of deduction (in our terminology, the syntactical rules of transformation) can be chosen arbitrarily and hence are conventional if they are taken as the basis of the construction of the language system and if the interpretation of the system is later superimposed. On the other hand, a system of logic is not a matter of choice, but either right or wrong, if an interpretation of the logical signs is given in advance. But even here, conventions are of fundamental importance; for the basis on which logic is constructed, namely, the interpretation of the logical signs (e.g., by a determination of truth conditions) can be freely chosen.

It is important to be aware of the conventional components in the construction of a language system. This view leads to an unprejudiced investigation of the various forms of new logical systems which differ more or less from the customary form (e.g., the intuitionist logic constructed by Brouwer and Heyting, the systems of logic of modalities as constructed by Lewis and others, the systems of plurivalued logic as constructed by Lukasiewicz and Tarski, etc.), and it encourages the construction of further new forms. The task is not to decide which of the different systems is "the right logic" but to examine their formal

properties and the possibilities for their interpretation and application in science. It might be that a system deviating from the ordinary form will turn out to be useful as a basis for the language of science.

III. Calculi and Their Application in Empirical Science

13. Elementary Logical Calculi

For any given calculus there are, in general, many different possibilities of a true interpretation. The practical situation, however, is such that for almost every calculus which is actually interpreted and applied in science, there is a certain interpretation or a certain kind of interpretation used in the great majority of cases of its practical application. This we will call the *customary interpretation* (or kind of interpretation) for the calculus. In what follows we shall discuss some calculi and their application. We classify them according to their customary interpretation in this way: logical calculi (in the narrower sense), mathematical, geometrical, and (other) physical calculi. The customary interpretation of the logical and mathematical calculi is a logical, L-determinate interpretation; that of the geometrical and physical calculi is descriptive and factual. The mathematical calculi are a special kind of logical calculi, distinguished merely by their greater complexity. The geometrical calculi are a special kind of physical calculi. This classification is rather rough and is only meant to serve a temporary, practical purpose.

To the logical calculi (in the narrower sense) belong most of the calculi of elementary structure used in symbolic logic, above all, the so-called sentential calculus and the so-called lower functional calculus. The *sentential calculus* has approximately the structure of B-C with F 4 and PS 4 omitted. The customary interpretation corresponds to the rules B-SL 2, 3. The form mostly used contains, however, only those signs which are logical in the customary interpretation, corresponding to the English words 'not', 'if', 'or', 'and', and the like, and sentential variables. The *lower functional calculus* (or predicate calculus)