Disjunction and Possibility

Matt Mandelkern

1. A puzzle: I argue for a novel generalization about disjunction and explore how to derive it. Suppose Latif is going to the store to buy one pie. He likes apple best, then blackberry, then cherry. The store has at least one of these, but may not have all three. Given this, there is a true reading of (1):

(1) If Latif buys apple or blackberry, he'll buy apple. $(A \vee B) > A$

This reading of (1) says, roughly: if Latif has the option of buying apple and of buying blackberry, he will choose apple. But the truth of (1) is still consistent with Latif buying blackberry, since the store might have blackberry but not apple. That is, (1) does not entail (2):

(2) Latif will not buy blackberry. $\neg B$

However, this is a puzzle, for given that Latif will buy one pie, (1) *does* entail (2) if we assume:

[*Or-Intro*] ⌜A or B⌝ is true if B is true; and

[*Modus Ponens*] if A is true and B is false, then ⌜If A, then B⌝ is not true.

Now suppose (1) is true at w, and suppose further that Latif buys one pie in w. Now suppose for contradiction that Latif buys blackberry in w. Since he buys only one pie in w, he doesn't buy apple in w. By *Or-Intro*, since B is true at w, so is $A \vee B$. By *Modus Ponens*, since $A \vee B$ is true and A is false, $(A \vee B) > A$ is false. But $(A \vee B) > A$ is just (1), which is true by assumption. So, given that Latif will buy one pie, (1) entails (2) given *Or-Intro* and *Modus Ponens*, which are standard assumptions. But, again, (1) clearly does not entail (2), on its prominent true reading (it may have another reading which does entail (2), but all that matters for us is that it has a true reading on which it does not). To bring this out, compare:

(3) If Latif buys apple or blackberry, he'll buy apple. But he might still buy blackberry, since apple might not be available.

(4) Latif won't buy blackberry. # But he might still buy blackberry, since apple might not be available.

If (1) entailed that Latif won't buy blackberry, these would feel equally infelicitous.

Cases like this are easy to multiply. Suppose Marie prefers to study abroad in Germany rather than France. Then it follows, on one prominent reading, that if she studies in Germany or studies in France, she will study in Germany. But we cannot conclude that she won't study in France, since it is consistent with this set-up that the German programs might be full, in which case Marie would go to France.

2. A puzzle about conditionals? The puzzle arises from *Or-Intro* and *Modus Ponens*, so we can consider in turn which principle we might reject. [7] argues we should reject *Modus Ponens*. However, McGee's argument only applies to conditionals whose consequent itself contains (overt) modals or conditionals. For other kinds of conditionals, McGee and just about everyone else agrees that *Modus Ponens* is still valid. Since our target conditionals' consequents do not contain modals or conditionals, this route is not helpful.[1]

3. A puzzle about disjunction? I suggest that my puzzle arises from the way we interpret the disjunction in the antecedent of (1). Evidence for this comes from a parallel phenomenon involving disjoined disjunctions, observed by [redacted], (p.c.). Suppose Marie has to choose from two programs, one with sites in France and Germany, and one with sites in France and Spain. Then intuitively (5) is assertable and true:

(5) Marie will go to France or Germany, or else she'll go to France or Spain.

However, given *Or-Intro*, (5) is equivalent to 'Marie will go to France or Germany or Spain', and hence on standard theories of redundancy (like [9, 8]), (5) should be unassertable. Intuitively, this is a parallel phenomenon, and I will suggest a unified solution. A further reason to think our puzzle is about disjunction comes from the contrast between (1) and (6):

(6) If Latif doesn't buy cherry, he'll buy apple. $\neg C > A$

Although 'Latif doesn't buy cherry' is contextually equivalent to 'Latif will buy apple or blackberry', (6) intuitively *does* entail that Latif won't buy blackberry, unlike (1). Intuitively, in all these cases, we enrich the interpretation of disjunctions to include possibility inferences. So, e.g., (1) is naturally glossed as (7):

(7) If *apple is possible* and *blackberry is possible* and Latif buys apple or blackberry, he'll buy apple.

(7) does not entail that Latif won't buy blackberry in the presence of *Or-Intro* and *Modus Ponens*. Hence, if we interpret (1) as (7), we dissolve our puzzle. Likewise, if a disjunction like (5) is interpreted with a possibility inference in each disjunct, as 'Either Marie will go to France or Germany and both those are possible, or she will go to France or Spain and both those are possible', then redundancy does not rule it out. In general, where $\blacklozenge$ is a possibility operator, what I take these puzzles to show is:

DIS-POS: A disjunction with the surface form 'A or B' can be interpreted as $(A \vee B) \wedge \blacklozenge A \wedge \blacklozenge B$.

$\blacklozenge$ must represent *circumstantial possibility* in particular. If it represented epistemic possibility, the resulting interpretation of (1) would be 'If Latif will buy apple or blackberry, and these are both epistemically possible, then he will buy blackberry'. This is, however, not strong enough to capture the intuitive meaning of (1), nor to get us out of the puzzle, since both apple and blackberry *are* epistemically possible.

While it is well-known since [4] that disjunction gives rise to *epistemic possibility inferences*, I do not know of any discussion of the fact that a disjunction can also give rise to corresponding *circumstantial possibility inferences*, making *DIS-POS* a novel and intriguing claim. ([5] discusses a simlar puzzle in the context of *belief updating*, but his solution depends crucially on the treatment of the belief-update operation, and hence is not general enough to explain facts about disjunction alone like the felicity of (5).)

[1] Our puzzle is superficially similar to the puzzle of *SDA*: accounting for the inference from $(A \vee B) > C$ to $(A > C) \wedge (B > C)$. But on reflection these puzzles are unrelated: *SDA* would let us infer from (1) that Latif will buy apple if he buys blackberry, but we clearly *don't* infer that. It has been observed that *SDA* fails precisely in the case of conditionals with the form $(A \vee B) > A$, but the reported judgment is that they entail $\neg B$, while our puzzle is precisely that this entailment appears to be missing.

5. Accounting for* DIS-POS*: I will very briefly reject a few natural accounts of *DIS-POS* before sketching a more promising approach. There are many possible approaches, and my goal is only to sketch one possibility.

A first thought is that *DIS-POS* is built into the meaning of 'or', so that 'A or B' just *means* $(A \vee B) \wedge \blacklozenge A \wedge \blacklozenge B$ (compare the proposal in [11], where the modality is epistemic). However, this has problems with negation, since 'Not (A or B)' entails 'Not A and not B', which doesn't hold on this proposal.

A different approach is to derive the target interpretation by inserting covert modals at LF (compare [6] on conditionals). However, it is not clear how this would work, since we need to transform 'A or B' to $(A \vee B) \wedge \blacklozenge A \wedge \blacklozenge B$. This transformation would require inserting not just a modal but rather a full clause, which would be deeply revisionary. A better idea is to instead insert a covert modal at LF which takes scope over $A \vee B$, and then *exhaustify* the result using the exhaustification operator along the lines of [2] (roughly, a covert version of 'only'). In particular, we say that 'A or B' has the logical form *EXH*$(\blacksquare(A \vee B))$, where $\blacksquare$ is the dual of $\blacklozenge$. This nicely yields the intended reading. (Exhaustifying without a covert modal would only get us epistemic possibility inferences, not the required circumstantial ones.) However, this overgenerates in a number of ways. E.g. it predicts a meaning for (6) where it is interpreted as $\blacksquare\neg C > A$, that is, 'If Latif *cannot* buy cherry, he will buy apple'. However, this reading is, as far as I can tell, unavailable.

To avoid this, I propose that circumstantial modals are added, *not* to the LF, but rather to the *alternatives* which are the input to the meaning of *EXH* (see again [2] for the formal details of *EXH*). That is, the alternatives to $A \vee B$ include not just A, B, and $A \wedge B$, as on standard accounts, but also $\blacklozenge A, \blacklozenge B, \blacklozenge(A \wedge B), \blacklozenge(A \vee B), \blacksquare A, \blacksquare B, \blacksquare(A \wedge B)$ and $\blacksquare(A \vee B)$ (for technical reasons I explain in the paper, both $\blacklozenge$ and its dual $\blacksquare$ are needed here). Then we exhaustify using the enriched algorithm from [1] (that is, with both *inclusion* and *exclusion*; alternately, we could exhaustify recursively as in [3]). Then *EXH*$(A \vee B)$ entails $\blacklozenge A \wedge \blacklozenge B$, so we get the desired strengthening. But *EXH*$(\neg C)$ does *not* entail $\blacksquare\neg C$, as desired, so we avoid overgenerating readings for (6), which still entails that Latif won't get blackberry, as desired. (*Or-Intro* fails for exhaustified disjunction on this theory, explaining how we avoid the puzzle at the start.)

6. In sum: The cases I have explored support *DIS-POS*: disjunctions give rise to circumstantial possibility implicatures. This is a novel generalization, and accounting for it is not straightforward. I develop one account by exhaustifying over scalar alternatives which are closed under circumstantial modals. There are many alternate approaches to explore, as well as further questions and possible extensions. One thing to explore is whether the $\blacklozenge$ operator here can be freely interpreted with different flavors of modality, in which case this algorithm would yield a new explanation of epistemic possibility inferences from disjunction. In addition, we should explore connections with free choice inferences, which are intuitively parallel, as well [5]'s puzzle about belief updating and related issues about confirmation in [10].

References

[1] Moshe E. Bar-Lev and Danny Fox. Free choice, simplification, and Innocent Inclusion. *Natural Language Semantics*, 28:175–223, 2020.

[2] Gennaro Chierchia, Danny Fox, and Benjamin Spector. The grammatical view of scalar implicatures and the relationship between semantics and pragmatics. In Paul Portner, Claudia Maienborn, and Klaus von Heusinger, editors, *Handbook of Semantics*. Mouton, 2012.

[3] Danny Fox. Free choice and the theory of scalar implicatures. unpublished manuscript, 2006.

[4] Paul Grice. Logic and conversation. In *Studies in the Ways of Words*. Harvard University Press, 1967/1989.

[5] Stephan Krämer. Mighty belief revision. *Journal of Philosophical Logic*, FC, 2022.

[6] Angelika Kratzer. The notional category of modality. In H. Eikmeyer and H. Rieser, editors, *Words, Worlds, and Contexts: New Approaches in Word Semantics*, pages 38–74. de Gruyter, 1981.

[7] Vann McGee. A counterexample to modus ponens. *The Journal of Philosophy*, 82(9):462–471, 1985.

[8] Philippe Schlenker. Be articulate: a pragmatic theory of presupposition projection. *Theoretical Linguistics*, 34(3):157–212, 2008.

[9] Raj Singh. *Modularity and Locality in Interpretation*. PhD thesis, Massachusetts Institute of Technology, Cambridge, MA, 2008.

[10] Stephen Yablo. Tangled up in grue. Handout for a talk in Geneva, June 2022.

[11] Thomas Ede Zimmermann. Free choice disjunction and epistemic possibility. *Natural Language Semantics*, 8(4):255–290, 2000.